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From the lines through (1, 1) we choose a set of four; to these add four through (2, 2) agreeing with them. This can be done in a number of ways. To the eight so found, it is impossible to add four through (3, 3) so that the fundamental assumptions hold. The first assumption breaks down, no matter what combination be tried.

There is therefore no Euclidean plane with 36 points; and consequently no projective plane with only seven points on a line. This includes the result for $n=7$ given in the answer to problem 142, p. 108, Vol. XIV. It is also the result given by Dr. F. H. Safford in answer to problem 132, p. 215, Vol. XIII.

ON CONSTRUCTING A CUBE HAVING A GIVEN RATIO TO A GIVEN CUBE.*

By R. D. CARMICHAEL.

The semi-cubical parabola is capable of a beautiful application to the problem of finding a cube having a given ratio to a given cube. The object of this paper is to give a method of constructing this curve by continuous motion, to apply the locus to the above problem, and also to show how to construct a line numerically equal in length to the cube root of a given line.

The equation of this curve,

$$(1) \quad x^3 = py^2,$$

when transformed to polar coordinates with the axis of x as the polar axis, may readily be reduced to

$$(2) \quad \rho \cos \theta = p \tan^2 \theta.$$

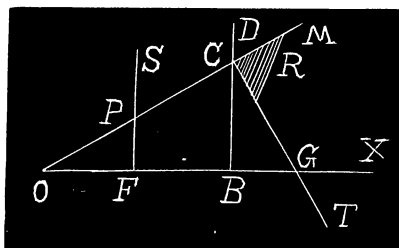


Fig. 1.

To find a construction of the curve by continuous motion we proceed as follows: Let OX (Fig. 1) be the polar axis. Take a distance $OB=p$ and erect BD perpendicular to OX , cutting OM at C , the angle MOX being equal to θ . Draw CG perpendicular to OM at C , intersecting OX in G . From O lay off $OF=BG$, and draw FS perpendicular to OX , intersecting OM in P . P is a point on the semi-cubical parabola.

Proof. Since $OB=p$, $BC=p \tan \theta$. But, since OCG is a right triangle and CB is perpendicular to OG , $OB:BC::BC:BG$. Hence,

*Read before the American Mathematical Society, April, 1907.

$$(3) \quad p.BG=BC^2=p^2\tan^2 \theta; \text{ or } BG=p\tan^2 \theta.$$

But by the equation of the curve, $p\tan^2 \theta=\rho \cos \theta$. But if $OP=\rho$, then $OF=\rho \cos \theta=p\tan^2 \theta=BG$. Hence P is a point on the locus.

Now since $OF=BG$ and $OB=p$, $FG=p$. The following construction of the curve by continuous motion is then evident: Let OM and OX be two material lines pivoted at O , OX being fixed to the plane of the paper. Let the material right triangle R , with the material straight line CG attached to it, be fastened to OM so that it may slide freely along OM . Let another material line $FG=p$ be attached to OX so as to slide along it and at the same time let CT pass through a ring at G of FG so as to slide freely. BC is a material line fastened to the plane of the paper at P and perpendicular to OX , OB being equal to p . It passes through a ring at the vertex of the right angle in the triangle R . Finally, FS is a material line fastened to GF perpendicular to it at its extremity F . This intersects OM at some point P . Then let a ring pass around OM and FS at P and let a pencil be placed in this ring. As OM revolves about O the pencil at P will describe the semi-cubical parabola $x^3=py^2$.

We shall now apply this locus to the problem of finding a cube having to a given cube the given ratio of the line m to the line n . For this we shall employ the special case $p=1$. In Fig. 2 lay off $x=OH=a$ side of the given cube. Draw $HQ=y$, Q being a point on the locus. Take $OB=-1$ and erect BC perpendicular to OX and cutting QR in C , QR being parallel to OX . Draw OM through C , and then CG perpendicular to OM at C and cutting OX in G . By the same reasoning as in finding equation (3) it follows that

$$BG=\overline{BC}^2,=\overline{QH}^2=y^2.$$

But $x^3=y^3$. Hence $BG=OH^3$.

Now find a fourth proportional to m , n , BG , and from B lay off this line, say BL . Extend BC to S and on OL as a diameter describe a semi-circumference cutting BS at W and through W draw a line parallel to OX and cutting the curve in T . From T let fall a perpendicular to OX at V . Then we may repeat the previous reasoning to show that $OV^3=BL$. And therefore, since $OH^3=BG$ and $m:n::BG:BL$, $OH^3:OV^3::m:n$.

We have seen above that $OH^3=BG$, $OH=\sqrt[3]{BG}$. We may thus construct a line numerically equal in length to the cube root of any other line; or, conversely, we may construct a line numerically equal to the cube of a given line.

Remark. It will of course be observed that the problem here solved is a generalization of that of the duplication of the cube, renowned from

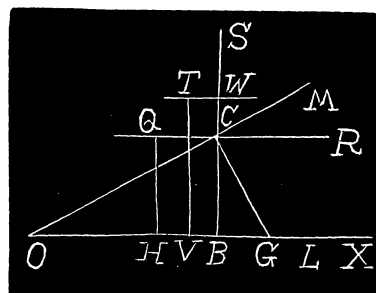


Fig. 2.

antiquity. By this method the ratio of the given cube to the required may be any capable of being expressed as the ratio of two straight lines, and that the problem is not more difficult of solution for one ratio than for another.

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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

GEOMETRY.

314. Proposed by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

Find the area of the triangle bounded by the lines $l^a + m^b + n^c = 0$; $l'a + m'\beta + n'\gamma = 0$; $l''a + m''\beta + n''\gamma = 0$, where a stands for $x \cos a + y \sin a - p$, etc. [See Salmon's *Conic Sections*, 6th Ed.]

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa., and C. N. SCHMALL, 89 Columbia Street, New York City.

$$\text{Let } A = a(mn' - m'n) + b(nl' - n'l) + c(lm' - l'm),$$

$$B = a(mn'' - m''n) + b(nl'' - n''l) + c(lm'' - l''m),$$

$$C = a(m'n'' - m''n') + b(n'l'' - n''l') + c(l'm'' - l''m').$$

Then the intersection of (1) and (2) is

$$\frac{\alpha_1}{mn' - m'n} = \frac{\beta_1}{nl' - n'l} = \frac{\gamma_1}{lm' - l'm} = \frac{2 \Delta}{A}.$$

The intersection of (1) and (3) is

$$\frac{\alpha_2}{mn'' - m''n} = \frac{\beta_2}{nl'' - n''l} = \frac{\gamma_2}{lm'' - l''m} = \frac{2 \Delta}{B}.$$

The intersection of (2) and (3) is

$$\frac{\alpha_3}{m'n'' - m''n'} = \frac{\beta_3}{n'l'' - n''l'} = \frac{\gamma_3}{l'm'' - l''m'} = \frac{2 \Delta}{C}.$$

The area of the triangle $(\alpha_1, \beta_1, \gamma_1)$; $(\alpha_2, \beta_2, \gamma_2)$; $(\alpha_3, \beta_3, \gamma_3)$ is

$$\frac{abc}{8\Delta^2} [\alpha_3(\beta_1\gamma_2 - \beta_2\gamma_1) + \beta_3(\gamma_1\alpha_2 - \gamma_2\alpha_1) + \gamma_3(\alpha_1\beta_2 - \alpha_2\beta_1)].$$